Mathematics Networks and Curriculum Concepts

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Student learning and performance in mathematics may be associated with issues of connectivity related to curriculum content, acting together with a larger and more complex set of issues, such as those related to teacher quality and socioeconomic influences. Complex and non-linear connectivity of mathematics and other concepts, in particular, may underpin the development of mathematics expertise, and student failures may be related to inadequate development of the networks that connect these concepts. This paper investigates the scope and influence of spatiotemporal networks that may link concepts in the learning of mathematics.

Mathematics has become a major component of curricula in institutionalised education in many industrialised societies, and is taught as a stand-alone subject with strong links to subjects allied with science, technology and economics. There have been concerns, however, that mathematics education is failing some students and that some education systems are not dealing well with mathematics over the broad school population (e.g., Haar et al., 2005). Australia can be seen as an illustrative case, where standards in mathematics, and interest in the subject, may be declining and where reports based on statistical analysis of national and international data, such as that gained in the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) surveys, have been disappointing in some areas when compared across the country or when compared across countries, both regionally in Asia and elsewhere (e.g., Leigh & Ryan, 2008). Additionally, Australian students may be having problems with mathematics at particular stages of their schooling, and this reflects problems apparent in other industrialised societies, for example, in the divergence of results during early secondary schooling in the USA (e.g., in California, see Kurlaender, Reardon, & Jackson, 2008).

Some studies, using approaches from the social and behavioural sciences in combination with science-based protocols, have suggested that problems in student performance, in mathematics at least, may be associated with issues of connectivity related to linkages of mathematics and other concepts (e.g., Lakoff & Núñez, 2000), issues that may act together with other issues, such as teacher quality and socioeconomic influences (e.g., Morrison, 2003). In particular, there have been attempts to link student learning and performance in mathematics to the complex and non-linear connectivity of concepts (e.g., Mowat and Davis, 2010) and, more specifically, to the growth of knowledge in connected pathways based on intrinsic neural networks, including networks related to emotions (e.g., Ellis, 2008). While there are other issues to be explored in the context of mathematics teaching in schools, including other issues that may involve complexity (e.g., see Morrison, 2003), or issues that relate to mathematics education more generally (e.g., Geary et al., 2011), this paper will consider a fundamental issue, the relationship of mathematics learning to curriculum content as perceived through assessment. The issue of what constitutes a mathematics concept and what is meant by the understanding of such a concept is also discussed, particularly in relation to concepts presented in a school mathematics curriculum. The main issue, here, however is the investigation of concepts as

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presented in the mathematics curriculum and how these concepts may be linked, both to each other and to concepts that are not in such curriculum, or not learned at school.

Concepts, mathematics and curriculum

Concepts and mathematics learning

Mathematics as a subject appears to be an artificial construct (e.g., Dehaene, 2007), with a lack of coherence or integrity due its development from a wide variety of sources and, arguably, its restrictive use in institutional education during industrialisation (e.g., OECD, 2003). Links between internal processes and external influences are apparent, however, in consideration of concepts related to the subject of mathematics. Berger (2005) argues Vygotsky's approach in observing that an individual's remembered mathematics concepts require a link to social knowledge, citing Ernest's (1997) framework of semiotic mediation as a mechanism of learning mathematics through acquiring mastery of the use of abstract sign systems. Regardless of how such concepts are acquired, once learned, mathematical concepts may be further modified through application to the real world, mediated by non-mathematical concepts related to real world situations as well as by a set of associated linguistic or symbolic expressions (Chassapis, 2010). A recent study (Tubbs, 2005) asserts that mathematics concepts should be treated as dynamic and intrinsic to almost every human endeavour, an issue related to concept integration that will be discussed later in this paper.

Studies in integrative biology support the engagement of environmental and internal interaction in learning mathematics concepts, although with a strong argument that suggests that societal expectation determines the categorisation of particular knowledge and skills as the subject of mathematics (e.g., Dehaene, 2007). Concepts that assist in the negotiation of our environment may, in fact, be co-opted for the abstracted information referred to as mathematics, with Lakoff and others (e.g., Lakoff & Núñez, 2000) arguing that concepts treated as mathematics, like other embodied concepts, are based in physical exploration of the world. There are of course arguments in the social sciences, such as those related to the views of Kant (see e.g., discussion Shabel, 1998), that mathematics is based on the construction of concepts in intuition that precede and structure how humans experience the environment, or that mathematical concepts exist regardless of any mental perceptions of them. Some scientists have suggested that the Kantian view, at least, may be an idealisation that may be replaced by a scientific understanding of underlying genetic and developmental mechanisms (Dehaene & Brannon, 2010), but, in any case, such arguments may not be relevant necessarily to the investigation of mathematics concepts as they are applied in a school mathematics curriculum.

Mathematics concepts, curriculum and assessment

Theoretical views of concepts, as interesting as they may be in academic circles, are not always recognised in a mathematics curriculum, a situation recognised by teachers. A mathematics curriculum in an industrialised society can be seen as a construct around which, hopefully useful, knowledge and skills are developed through learning conducted in an institutional setting, and the modern mathematics curriculum is, in effect, a learned curriculum (e.g., see Chapter 1 in Glatthorn et al., 2012). The level of understanding may vary, but teachers depend on assessment, usually written assessment, to determine any success that a student may have had in learning what may be determined as a concept. In this sense the notion of a concept can be directly related to what is being taught and then assessed, regardless of the educational theory or practice that is being employed. It is this notion that enables a primary school teacher to consider the addition of two numbers less than ten to be a concept, but perhaps later to consider the generalisation of addition of any two numbers to be also a concept. If a mathematics concept being taught is being defined in terms of what the learner can do with it, Devlin (2007) has argued that the learner may be considered as having a functional understanding of that concept, an understanding that enables the learner to be tested for some level of understanding of that concept. He argues that, like a beginning chess player, a beginning maths student does not need to understand mathematics fully in order to use simple concepts and rules, so that a functional understanding of a concept may be a long way from a full understanding of that concept in the broader field of mathematics.

Mathematics curriculum and links between concepts

The view of a mathematics concept as determined by a curriculum and its assessment, however simplistic, is useful in that the links between concepts may be traceable using assessment results, even if this is only the tracing of a functional understanding. It may be possible, for example, to determine if a student who has answered successfully a question related to properties of a circle has knowledge also of other mathematics concepts that have led either linearly, or through a network of supporting links, to the knowledge of a circle. It is the example of a circle, in fact, that is used in Mowat and Davis (2010) to illustrate the potential network of linked concepts, rather than linear linkages, that may be involved in the learning of a mathematics concept. In this paper the argument is accepted that a concept can be built from other concepts in the style of schema building and considers as a parallel notion the building of a concept through the growth of connections in a neuronal network. This is, in part, to avoid the use of the notion of a metaconcept and any confusion in this discussion that the use of this and other such hierarchical terms may cause.

The exploration of networks and their linkages, an extension of graph theory, has developed rapidly in recent years and has been applied widely in a number of differing disciplines, largely because the rules governing the relationships within such networks remain independent of the nature of the subjects being linked (e.g., Watts, 2003). The main focus of the study of Mowat and Davis (2010) was, in fact, an argument that mathematics could be integrated through an examination of the complex linkages between mathematics concepts treated as the embodied metaphors of Lakoff and Núñez (2000), but a sidebar to this argument is that mathematics concepts so integrated must be linked as networks, as illustrated in the example of a circle. This argument seems to have support from the notion of expertise gained through the development of schemas, themselves arguably a type of network, as well as the notion from integrative biology that remembered knowledge and skills are a result of networked linkages grown, sometimes preferentially from other networks, within the nervous system (e.g., Sporns, 2010).

As Khattar (2010) has pointed out, the development of complex networks of conceptual mathematics knowledge may be dependent, therefore, on the development of hubs in the networks that support such conceptual development. It follows that student failures in mathematics may be related specifically to the lack of adequate development of multiple hubs within the complex linkages or networks that connect these concepts. Not only does this mean that mathematics concepts may be linked together, but also that they must be linked to other concepts in what Khattar (2010) considers to be essentially bodily experiences that are experienced emotionally. This view is reflected in notions of remembering from integrative biology that argue for memories as all of our input information, stored in long-term memory, automatically categorised in hierarchical networks (Mottron, Dawson, & Soulières, 2009). Mathematics education, from this point of view, may need to engage particular conceptual links and inhibit others, as in the notion of Snyder and colleagues (e.g., Snyder et al., 2004), since the linear nature of algorithms may rely on non-linear representations internally that are complex organisations of patterns.

Examining the scope and influence of links between concepts

The scope and influence of spatiotemporal networks has been under investigation in studies where these networks are seen operate as connections between structural and functional parts of the brain and nervous system, both in humans and other animals (e.g, Sporns, 2010). Sporns and others (e.g., Calvin, 2004; Sporns, 2010) have argued that these networks form the basis, in fact, of all of the interactions involved in human learning and memory, and this all-encompassing scope, therefore, matches their influence in determining the negotiation of the organism in its environment. Although the activity of neural networks may be correlated with observations of behaviour and with post-experimental observations, there are arguments that this type of research in integrative biology is not causal evidence for these corresponding observations and that the relationship between cognitive psychology and integrative biology remains unclear (e.g., Miller, 2010). There are also problems with observations and interpretations of neurobiological data that may affect such correlations (e.g., Rousselet & Pernet, 2012), and there are clearly some broader problems also with the use of neuroscience data in studies in education (Dekker et al., 2012).

In an educational context, although some educational theories have included the notion of complex networks (e.g., Davis, Sumara, & Luce-Kapler, 2008), there has been little development of the concept of conceptual linkages within a curriculum. Complexity theory, however, is being used to suggest reconstructions of curriculum, even though such change may run counter to the current outcome-driven education that uses the present curriculum structures (e.g., Morrison, 2003). What both Khattar (2010) and Mowat and Davis (2010) appear to be implying, in contrast, is that it may be possible to adjust the focus within the current mathematics curriculum so that hubs are supported by the development of redundancies through the introduction of what are referred to as weak links that create supporting networks around the hubs. These supporting networks mean that the collapse of a hub may not necessarily lead to the cascading failure of network function that is typical when a hub is compromised. Removal of a weak link will not cause such failure, but insertion of such a link may offer an alternative network that may function without the compromised hub.

Varma and Schwartz (2008) have presented arguments that the co-functioning of linkages, as networks between areas of the brain, may be of paramount importance in human learning and memory of concepts, and holistic views have argued also for the involvement also of networks outside the nervous system (e.g., Squire & Kandel, 2008; Woolcott, 2011). In the view of Varma and Schwartz (2008), teaching mathematics involves often complicated or multifaceted concepts and the examination of such concepts as they are involved in memories in the nervous system, therefore, requires a network focus, a focus that may allow consideration also of networks for well-learned tasks that are different to those used in initial learning stages. It may be important, therefore, to examine the scope and influence of these networks in relation to curriculum, in particular to the learned curriculum (in the sense of Glatthorn et al., 2012).

With regard to the concepts in the learned mathematics curriculum, however, here has been little work on the relationship of internal networks to external concepts except in areas related to number or arithmetic. Neural networks, for example, have been found to link causally to the performance of arithmetic tasks (e.g., Ansari et al., 2005) and different arithmetic operations, such as subtraction and multiplication, have been found to operate using weighted components of common networks (e.g., Duffau et al., 2002). Memorisation of mathematics facts has been found to utilise different networks to those used in learned algorithms where both have been aimed at answering the same multiplication questions (e.g., Delazer et al., 2005). Any evidence of networks that operate outside of number, and even those that operate in solving number problems, however, seem to indicate that the networks are not only complicated, but also that these networks may be shared across subject domains (e.g., Dehaene, 2007). Some researchers have argued for the integration of mathematics concepts through activities that yield coordinated networks (Case et al. 1997), and this integration ties the development of concepts back to the semiotic and holistic views mentioned earlier in this paper. Such holistic views, however, imply that, since neural and linked behavioural networks may be shared networks, there may need to be the focus in education on developing integrated networks that are independent of curriculum boundaries. This may be a logical follow-on from the views of such researchers as Mowat and Davis (2010), where the networks that support a mathematics concept, such as the circle, may also support learning of concepts in other subjects and may, in fact, be learned in other subjects.

Conclusion

The scope and influence of networks may be all-encompassing in human learning and memory, and the influence of studies that use networks is beginning to be felt in education, requiring new perspectives and new theories and practices that support the growth of internal networks in students so that the external networks of concepts may be supported. The scope and influence of networks in the learned curriculum and, indeed, in other views of curriculum, is only now starting to be investigated and in the mathematics curriculum this investigation has started in the area of number or arithmetic (e.g., Ansari et al., 2005; Varma & Schwartz, 2008)). One way to expand this investigation would be to examine student assessment, since this is a conventional method of determining the success of any teaching of a curriculum. In mathematics education, this is timely, since there may be problems occurring in the singular application of linear and hierarchical views of education to the attainment of mathematics knowledge and skills (Khattar, 2010; Mowat & Davis, 2010). A closer examination of student assessment results both spatially and longitudinally in a number of differing subjects, but including mathematics, may indicate where conceptual linkages are occurring and where these linkages may cross into other subject domains. This may indicate where those networks may require intervention, for example, through the development of weak links to support hubs. There are a number of studies that have urged the consideration of curriculum integration in terms of a broadly contextualised curriculum (e.g., Dehaene, 2007) and this, in fact, may be part of the solution to the growing of networks that have inbuilt redundancies and which recognise Tubbs (2009) view that mathematics may be present in some way in all areas of human life.

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